Quiz 4 Solutions

1. Recall Newton's Method says

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Since $f(x) = x^3 - 5$, $f'(x) = 3x^2$

$$\therefore \quad x_{n+1} = x_n - \frac{x_n^3 - 5}{3x_n^2}$$

$$= \frac{3x_n^3 - x_n^3 + 5}{3x_n^2}$$

$$\underbrace{x_{n+1} = \frac{2x_n}{3} + \frac{5}{3x_n^2}}$$

Initial guess: $x_0 = 1$ (choices may vary here)

$$x_1 = \frac{2}{3} + \frac{5}{3} = \frac{7}{3}$$

$x_2 \approx 1.86167800$

$x_3 \approx 1.72200188$

$x_4 \approx 1.71005974$

$x_5 \approx 1.70997595$

$x_6 \approx 1.70997595$

$\vdots$

To 5 places,

$$\boxed{\sqrt[3]{5} \approx 1.70998}$$

Check: $1.70998^3 \approx 5.000036.$

2. $f(x) = \dfrac{x^2 - 4}{x^2 + 4}$

a) Quotient rule:

$$f'(x) = \frac{(x^2+4)(2x) - (x^2-4)(2x)}{(x^2+4)^2}$$

$$= \frac{2x\,(x^2+4-x^2+4)}{(x^2+4)^2}$$

$$\boxed{f'(x) = \frac{16x}{(x^2+4)^2}}$$

$$f''(x) = \frac{(x^2+4)^2 \cdot 16 - 16x \cdot 2(x^2+4) \cdot 2x}{(x^2+4)^4}$$

$$= \frac{(x^2+4)\cdot 16 - 64x^2}{(x^2+4)^3}$$

$$\boxed{f''(x) = \frac{64 - 48x^2}{(x^2+4)^3}}$$

b) Critical point: $x = 0$ only.

$$f''(0) = \frac{64}{4^3} = 1 > 0$$

$\Rightarrow$ $\boxed{x = 0 \text{ Local minimum by } 2^{nd} \text{ Derivative Test}}$

c) Since $(x^2+4)^2 > 0$ for all x, $f'(x) > 0$ if and only if $16x > 0$, if and only if $x > 0$. Else; $f'(x) < 0$.

$\boxed{\begin{array}{ll} \text{Increasing}: & (0, \infty) \\ \text{Decreasing}: & (-\infty, 0) \end{array}}$

d) Again, use $x^2 + 4 > 0$ for all x, so we just study the sign of $64 - 48x^2$.

$$64 - 48x^2 = 0 \Rightarrow 4 - 3x^2 = 0$$

$$\Rightarrow x^2 = 4/3$$

$$\Rightarrow x = \pm \frac{2}{\sqrt{3}}$$

Sign chart:

$-$ $\quad$ $+$ $\quad$ $-$

$-2/\sqrt{3}$ $\quad$ $+2/\sqrt{3}$

Since $64 - 48(\pm 2)^2 < 0$, $64 - 48(0)^2 > 0$.

Concave up: $(-2/\sqrt{3}, 2/\sqrt{3})$

Concave down: $(-\infty, -2/\sqrt{3}) \cup (2/\sqrt{3}, \infty)$

e) $\lim_{x \to \pm\infty} \dfrac{x^2 - 4}{x^2 + 4} = 1 \longrightarrow$

$y = 1$ horizontal asymptote

No oblique / vertical asymptotes

$$\boxed{\text{Intercepts: } (0, -1), \quad (\pm 2, 0)}$$

f) Careful plot of f: